Duopoly Competition in Time-Dependent Pricing for Improving Revenue of Network Service Providers

Cheng ZHANG†, Student Member, Bo GU†‡, Member, Kyoko YAMORI†††, Senior Member, Sugang XU†§, Member, and Yoshiaki TANAKA††††, Fellow

SUMMARY Due to network users’ different time-preference, network traffic load usually significantly differs at different time. In traffic peak time, network congestion may happen, which make the quality of service for network users deteriorate. There are essentially two ways to improve the quality of services in this case: (1) Network service providers (NSPs) over-provision network capacity by investment; (2) NSPs use time-dependent pricing (TDP) to reduce the traffic at traffic peak time. However, over-provisioning network capacity can be costly. Therefore, some researchers have proposed TDP to control congestion as well as improve the revenue of NSP. But to the best of our knowledge, all of the literature related time-dependent pricing scheme only consider the monopoly NSP case. In this paper, a duopoly NSP case is studied. The NSPs try to maximize their overall revenue by setting time-dependent price, while users choose NSP by considering their own preference, congestion status in the networks and the price set by the NSPs. Analytical and experimental results show that the TDP benefits the NSPs, but the revenue improvement is limited due to the competition effect.

key words: time-dependent pricing, revenue maximization, duopoly competition

1. Introduction

The huge growth in demand for broadband data [1] is forcing Network Service Providers (NSPs) to use pricing as a congestion management tool. This trend is evidenced by the adoption of usage-based data pricing instead of the traditional flat-rate data plan by the major wired and wireless NSPs in US, Europe and so on [2]–[5]. However, the usage-based pricing could not solve the congestion problem at a given time without giving network users time-dependent incentives when the congestion happens, as Andrew Odlyzko et al. wrote in [5].

Unless UBP (Usage-Based Pricing) contains a time-of-day billing feature and some immediate feedback on congestion, it is hard to imagine how it can be used as a congestion management tool.

Previous works have shown that time-dependent pricing (TDP) can give the network users right incentive to shift their traffic demands when the network get congested [6]–[8]. MacKie-Mason and Varian [6] present the idea of a “Smart Market”, in which network adapts the prices to congestion levels. Each user places a “bid” on each packet that reflects his/her willingness to pay to send the packets on to the network at a given time. The packet is admitted if the bid exceeds the current cutoff amount, which is determined by the marginal congestion costs. Although the real-time feature of “Smart Market” is attractive in terms of efficiency, many issues still need further investigation, including accounting complexity, users’ burden of decision making and the implementation cost. Ha et al. proposed a time-dependent pricing scheme for mobile data communication, which gives users monetary reward to delay traffic during traffic peak time [7]. Unlike the real-time feature of the “Smart Market” in [6], time is slotted in [7], such as 48 time slots for one day, 30 minutes per slot. They conducted surveys which revealed that users are indeed willing to wait 5 minutes (for YouTube videos) to 48 hours (for software updates). They concluded that the time-dependent pricing flattens temporal fluctuation of traffic usage and benefits both users and NSP. Jiang et al. [8] studied hourly time-dependent pricing offered by a monopoly selfish NSP, comparing the profit-maximizing time-dependent prices to the socially optimal ones in the case of complete information and incomplete information with users’ utilities. Although the congestion effects were taken into account in [8], the competition between NSPs were not studied.

As written in [5] by Andrew Odlyzko et al., not only congestion, but also competition plays an important part on the NSPs’ pricing strategy. Different from above papers [6]–[8], in this paper, both competition between NSPs and congestion effect are taken into account in the case of incomplete information of users’ willingness to pay (WTP). A duopoly competition is studied in this paper, in which two NSPs set different pricing strategies to maximize their revenue.

Duopoly competition is widely studied in economics literatures in the form of Cournot competition and Bertrand competition [9]–[11]. In Cournot competition, different firms strategically choose quantities independently at the same time, while prices are determined in the markets to equate demand with the chosen quantities. Meanwhile in the Bertrand competition, different firms strategically choose

DOI: 10.1587/transcom.E96.B.2964

Copyright © 2013 The Institute of Electronics, Information and Communication Engineers
prices independently at the same time while supplying quantities demanded at the chosen prices. In the network economics area, competition is also studied in many literatures with considering the characteristic of telecommunication networks. Jin et al. [12] studies the competition between incumbent and emerging network technologies with the consideration of negative network externality [13]. Gibbens et al. [14] studies the duopoly competition between two NSPs with the consideration of negative network externality [13] when the NSPs differentiate their services. However, the prices set by the NSPs in [12], [14] are not time-dependent. D. Acemoglu and A. Ozdaglar in [15] studied the oligopoly competition between NSPs with considering the congestion costs that users imposed on others, and studied efficiency loss in terms of social welfare. However, an implicit assumption in [15] is that users are homogeneous in the sense that their valuations of Quality of Service (QoS) are the same.

Different from the aforementioned papers [12], [14], [15], in this paper, the prices set by the duopoly NSPs are time-dependent and the users valuations of QoS are heterogeneous, which means that different users may have different valuation on the same level of QoS.

The main contributions of this paper are as follows:

Firstly, the impact of NSP competition on the NSP revenue maximization is analyzed for duopoly case. In each time slot, we model the NSP duopoly competition as a Bertrand competition (price competition) game, in which each NSP sets price to compete for market share (number of users) to maximize its revenue. The sufficient condition for the existence of Nash equilibrium is established. The unique Nash equilibrium is also established under the assumption that the users’ valuation of QoS is uniformly distributed.

Secondly, heterogeneous users’ valuations of QoS and the congestion effect are also modeled, which is much more realistic.

The rest of this paper is organized as follows. NSP model and user model are presented in Sect. 2 and Sect. 3, respectively. In Sect. 4, the revenue maximization problem is formulated, then the Nash equilibrium of the Bertrand competition game is established for NSPs to choose the time-dependent pricing strategy in each time slot. Numerical results are presented in Sect. 5. Section 6 concludes this paper.

2. NSP Model

Consider a communication market with two NSPs, denoted by $S_1$ and $S_2$, respectively. NSP $S_1$ and $S_2$ provide substitute network services to network users. There exists a sequence of time, i.e., $t = 1, 2, ..., T$, at which each NSP sets time-dependent price $p_i^t$ (where $i = 1$ or 2). It is assumed that the population of users denoted by $N$ is fixed, with $N_i^t$ as the number of users choosing NSP $S_i$ for $i = 1$ or 2 at time slot $t$. The proportion of users who choose NSP $S_i$ at time $t$ is denoted by Eq. (1) as presented in [12].

\[ x_i^t = \frac{N_i^t}{N}, \text{ where } i = 1 \text{ or } 2 \]  

(1)

It is assumed that the value of $N$ is very large, either of the NSP cannot accommodate all the $N$ users.

The following set $D^t$ defined in Eq. (2) is the domain for $x_1^t$ and $x_2^t$.

\[ D^t = \{(x_1^t, x_2^t) | x_1^t + x_2^t \leq 1, 0 \leq x_1^t \leq 1, 0 \leq x_2^t \leq 1\} \]  

(2)

The quality of service (QoS) provided by the NSP $S_i$ for $i = 1$ or 2, denoted as $q_i$, is assumed decreased with the number of its subscribers due to the congestion. We employ a function $h_i(\cdot)$ defined on $[0, 1]$ to express the QoS provided by NSP $S_i$ at time slot $t$ as $q_i = h_i(x_i^t)$. The following assumption is for the QoS function $h_i(\cdot)$.

Assumption 1: $h_i(\cdot)$ is a non-increasing and continuous differentiable positive function of the number of users in network $S_i$ for $i = 1$, or 2. Both NSP $S_1$ and $S_2$ provide best effort service. Without loss of generality, the QoS provided by NSP $S_1$ is greater than that provided by NSP $S_2$.

Remark 1: The assumption of function $h_i(\cdot)$ captures the congestion effects that users experience when choosing NSP $S_i$ with limited resources [16]–[18] Ren et al. adopted the same QoS assumption when considering the QoS formulation of an entrant NSP in a Femtocell communication market [18], and on the other hand assumed that the QoS of an incumbent NSP in Femtocell communication markets is constant. Our analysis differs from [18] mainly in three ways: (i) two incumbent NSPs are considered; (ii) the QoS of both NSPs is decreased with the number of the users in their respective network; and (iii) the prices set by NSPs are time-dependent.

3. User Model

A continuum model of users is employed in this paper. If there are a large number of users in the communication market and each individual user is negligible, the continuum model approximates well the real user population [8]. The payoff of user $k$ at time $t$ is denoted as Eq. (3)

\[ u_{k,i}^t = \theta_k^t q_i^t - p_i^t \]  

(3)

where $\theta_k^t \in [0, \varphi^t]$ is the QoS valuation of user $k$ at time slot.
At each decision-making time $t$. Please note that $\theta_k^i$ is time-dependent, reflecting users’ different preference for different time slot [8]. The value of $\theta_k^i$ is private information of users, but the distribution of $\theta_k^i$ is public information of NSPs. Furthermore, different users may have different valuations on the same level of QoS. $q_k^i$ denotes the QoS provided by NSP $S_i$’s network, and $\theta_k^i$ denotes the benefit that the user can get from NSP $S_i$. The unit of user’s valuation of QoS (i.e., $q_i^k$) is chosen such that $\theta_k^i q_k^i$ has the same unit with that of the payment $p_t^i$, which is the price set by NSP $S_i$ at each time slot. We have the following assumption for users’ valuations of QoS.

**Assumption 2:** The users’ valuations of QoS have the probability density function (PDF) $f(t)$, which is strictly positive and continuous on $[0, \varphi^t)$ for $\varphi^t > 0$. The cumulative density function (CDF) is defined by $F(a) = \int_{-\infty}^{a} f(y)dy$ for all $a \in R$.

**Remark 2:** It is assumed that the NSP has incomplete information with the users’ valuations of QoS [19], which is much more reasonable compared with the assumption of having complete information with users’ valuation of QoS [20]. The lower bound of the domain of the probability density function is set as zero to simplify the analysis. NSPs need to know the distribution of the users’ valuations of QoS by conducting market surveys and using data mining and learning techniques. The details of information acquisition are beyond the scope of this paper.

At each decision-making time $t$, each user only chooses one NSP’s network. Each user is a rational decision maker, which means that (1) individual-rationality constraint and (2) incentive-compatibility constraint should be satisfied.

Individual-rationality constraint means that each user chooses the NSP $S_i$ only if he/she gets positive payoff by using NSP $S_i$. Incentive-compatibility constraint means that each user choose the NSP who can provide a relative higher payoff to him/her. In other words, a user chooses an NSP under the conditions enumerated as follows.

**Assumption 3:** There is no switching cost when users change from one NSP to another. At each decision time $t$, each user makes decision independently.

**Remark 3:** Although it is technically feasible for the users to change their NSP at anytime, users always face switching cost when they change their NSP in real world. We make further comment on this assumption in the conclusion section.

The notations used throughout this paper are summarized as shown in Table 1.

### 4. Revenue Maximization

The NSPs try to maximize their overall revenue by maximizing their revenue in each time slot. We model the NSP duopoly competition as a Bertrand competition game for each time slot. Unique Nash equilibrium of the Bertrand competition game is established under the assumption that the users’ valuation of QoS is uniformly distributed. The revenue of the NSPs depends on the number of users in the network and the prices set by the NSPs, which is denoted as $R_t = p_t^1 x_t^1(p_t^1, p_t^2)$. Please note that the number of user in network $S_1$, $x_t^1(p_t^1, p_t^2)$, is a function of the price set by network $S_1$ and $S_2$, which can be calculated by the Proposition 1 established in this section. The overall revenue of each NSP can be expressed as $\sum_{t=1}^{T} R_t$.

**Assumption 4:** The level of QoS at the beginning of time slot $t$ is estimated from the number of users in the end of time slot $t - 1$.

**Remark 4:** Since that we have assumed both NSP $S_1$ and $S_2$ provide best effort service and the number of users changes with time, the level of QoS also changes with the time. It is very complicated to consider the real-time QoS, and this assumption is based on “If there is a large number of users in the last time slot, and the QoS is bad in the last time slot, then expectation of QoS in the current time slot is also bad”. The level of QoS at the beginning of time slot $t$ can be expressed as $h_1(x_t^{-1})$. This kind of estimation is valid in that if there is large number of users in the network, the marginal effect of a single user’s impaction on QoS of the whole network can be neglected. But the accumulation effect of many users’ impaction on QoS cannot be neglected. As shown in Proposition 1, the accumulation effect of many users’ impaction on QoS make the number of users evolve as $\Omega_1(x_t^{-1})$ and $\Omega_2(x_t^{-1})$.  

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$i \in {1, 2}$, which is NSP set</td>
</tr>
<tr>
<td>$k$</td>
<td>subscript of a user</td>
</tr>
<tr>
<td>$S_i$</td>
<td>the two duopoly NSP in the market, for $i = 1, 2$</td>
</tr>
<tr>
<td>$T$</td>
<td>the total time slots</td>
</tr>
<tr>
<td>$t$</td>
<td>$t \in {1, T}$, the $t$-th time slot</td>
</tr>
<tr>
<td>$N$</td>
<td>the population of users</td>
</tr>
<tr>
<td>$N_i^t$</td>
<td>the number of users choose NSP $S_i$ for $i = 1, 2$ at time $t$</td>
</tr>
<tr>
<td>$x_i^t$</td>
<td>$x_i^t = N_i^t / N$: the proportion of users who choose NSP $S_i$ at time $t$</td>
</tr>
<tr>
<td>$\theta_k^i$</td>
<td>user $k$’s valuation of QoS at time slot $t$</td>
</tr>
<tr>
<td>$p_t^i$</td>
<td>the set by NSP $S_i$ at time slot $t$</td>
</tr>
<tr>
<td>$\varphi^t$</td>
<td>the upper bound of the domain of the function $f(t)$</td>
</tr>
<tr>
<td>$\Omega(t)$</td>
<td>cumulative density function (CDF) of users’ valuation of QoS at time $t$</td>
</tr>
<tr>
<td>$\varphi^t$</td>
<td>the marginal point where users switch from getting negative payoff to deriving positive payoff from choosing NSP $S_1$</td>
</tr>
<tr>
<td>$\tau_i^t$</td>
<td>the marginal point where users switch from using NSP $S_2$ to using NSP $S_1$</td>
</tr>
<tr>
<td>$R_t^i$</td>
<td>the revenue get by NSP $S_i$ at time slot $t$</td>
</tr>
</tbody>
</table>
At time slot $t$, user $k$ chooses NSP $S_1$ if and only if the conditions shown in inequality (4) are satisfied.

$$\theta_k h_1(x_1^{t-1}) - p_1^t \geq \theta_k h_2(x_2^{t-1}) - p_2^t \text{ and } \theta_k h_1(x_1^{t-1}) - p_1^t \geq 0 \quad (4)$$

Similarly, user $k$ chooses NSP $S_2$ if and only if the conditions shown in inequality (5) are satisfied.

$$\theta_k h_2(x_2^{t-1}) - p_2^t \geq \theta_k h_1(x_1^{t-1}) - p_1^t \text{ and } \theta_k h_2(x_2^{t-1}) - p_2^t \geq 0 \quad (5)$$

Otherwise, user $k$ chooses neither of NSP $S_1$ and $S_2$ if and only if the conditions shown in inequality (6) are satisfied.

$$\theta_k h_1(x_1^{t-1}) - p_1^t < 0 \text{ and } \theta_k h_2(x_2^{t-1}) - p_2^t < 0 \quad (6)$$

Now we characterize the marginal points that identifying user's valuation of QoS associated with changes in their decision to choose either NSP. $\tau^0_{S_1}$ denotes the marginal point where users switch from getting negative payoff to deriving positive payoff from choosing NSP $S_1$, i.e., $\tau^0_{S_1}$ is a point such that $u'_{k,1} = 0$. Similarly, $\tau^0_{S_2}$ corresponds to the marginal point where users switch from using NSP $S_2$ to using NSP $S_1$, i.e., $\tau^0_{S_2}$ is the point such that $u'_{k,2} = 0$. With the definition of $\tau^0_{S_1}$ and $\tau^0_{S_2}$, we can have the following,

$$u'_{k,1} = \theta_k h_1(x_1^{t-1}) - p_1^t > 0 \text{ if } \theta_k > \tau^0_{S_1} \quad (7)$$

$$u'_{k,2} > u'_{k,2} \text{ if } \theta_k > \tau^0_{S_2} \quad (8)$$

Eq. (7) indicates that if $\theta_k > \tau^0_{S_1}$, then the user with QoS valuation greater than $\tau^0_{S_1}$ can get positive payoff from choosing NSP $S_1$. Equation (8) indicates that the user with QoS valuation larger than $\tau^0_{S_2}$ will choose NSP $S_1$ since he/she can get greater payoff from NSP $S_1$ than from NSP $S_2$. Therefore, it is very important to compute these marginal points, which determine the users' choice of NSP. Please note that although $\tau^0_{S_1}$ and $\tau^0_{S_2}$ are also time-dependent, $t$ is not written in the expression of $\tau^0_{S_1}$ and $\tau^0_{S_2}$ for clear concern.

By setting $u'_{k,1} = 0$, we can derive $\tau^0_{S_1}$ and $\tau^0_{S_2}$ as shown in Eq. (9) and Eq. (10), respectively.

$$\tau^0_{S_1} = \frac{p_1^t}{h_1(x_1^{t-1})} \quad (9)$$

$$\tau^0_{S_2} = \frac{p_2^t}{h_2(x_2^{t-1})} \quad (10)$$

By setting $u'_{k,2} = u'_{k,2}$, we can derive $\tau^1_{k}$ as shown in Eq. (11).

$$\tau^1_{k} = \frac{p_1^t - p_2^t}{h_1(x_1^{t-1}) - h_2(x_2^{t-1})} \quad (11)$$

Lemma 1: If $\frac{p_1^t}{h_1(x_1^{t-1})} < \frac{p_2^t}{h_2(x_2^{t-1})}$, then $\tau^1_k < \tau^0_{S_1} < \tau^0_{S_2}$. If $\frac{p_1^t}{h_1(x_1^{t-1})} \geq \frac{p_2^t}{h_2(x_2^{t-1})}$, then $\tau^1_k \geq \tau^0_{S_1} \geq \tau^0_{S_2}$.

Proof. Please refer to Appendix A for the details of proof.

Now, we consider the following two subsets of users who have valuations of QoS defined in $[0, \varphi^*]$.

$$\Theta_1(x') = \left\{ \theta_k^t \in [0, \varphi^*] | u'_{k,1} \geq u'_{k,2}, u'_{k,1} > 0 \right\} \quad (12)$$

$$\Theta_2(x') = \left\{ \theta_k^t \in [0, \varphi^*] | u'_{k,2} > u'_{k,1}, u'_{k,2} > 0 \right\} \quad (13)$$

Eq. (12) defines QoS valuation of users who choose NSP $S_1$. Equation (13) defines QoS valuation of users who choose NSP $S_2$. We denote the number of users in each set at time slot $t$ as $\Omega(x')$.

**Proposition 1:** For any non-negative price pair $(p_1^t, p_2^t)$, the number of users in NSP $S_1$ and $S_2$’s networks are presented in Eq. (14) and Eq. (15), respectively.

$$\Omega_1(x') = \left\{ \begin{array}{ll}
1 - F^*(\tau^0_{S_1}) & \text{if } \frac{p_1^t}{h_1(x_1^{t-1})} < \frac{p_2^t}{h_2(x_2^{t-1})} \\
1 - F^*(\tau^0_{S_2}) & \text{if otherwise}
\end{array} \right. \quad (14)$$

$$\Omega_2(x') = \left\{ \begin{array}{ll}
0 & \text{if } \frac{p_1^t}{h_1(x_1^{t-1})} < \frac{p_2^t}{h_2(x_2^{t-1})} \\
F^*(\tau^0_{S_2}) - F^*(\tau^0_{S_1}) & \text{if otherwise}
\end{array} \right. \quad (15)$$

**Proof.** Please refer to Appendix B for the details of proof.

$p_i^{t'}$ is the price per QoS of NSP $S_i$ at the beginning of time slot $t$. If $\frac{p_1^{t'}}{h_1(x_1^{t-1})} < \frac{p_2^{t'}}{h_2(x_2^{t-1})}$, it means that the price per QoS of NSP $S_1$ is lower than that of NSP $S_2$. In this case, the number of users who choose NSP $S_1$ is positive, but the number of users who choose NSP $S_2$ is zero. The reason why the market share of NSP $S_2$ is zero in this case is that the NSP $S_1$ always provides better QoS than that of NSP $S_2$ (see Assumption 1). $\frac{p_1^{t'}}{h_1(x_1^{t-1})} < \frac{p_2^{t'}}{h_2(x_2^{t-1})}$ also means that the NSP $S_2$ with worse QoS charges a much higher price. Therefore, rational users would not choose NSP $S_2$. If $\frac{p_1^{t'}}{h_1(x_1^{t-1})} \geq \frac{p_2^{t'}}{h_2(x_2^{t-1})}$, it means that the price per QoS of NSP $S_2$ is lower than that of NSP $S_1$. Both NSP $S_1$ and $S_2$ have positive number of users who use their networks. This proposition shows that the price per QoS rather than the price determines the market share of a NSP. When an NSP set its price, the competitor’s price per QoS should be considered to keep its network competitive.

Each NSP tries to maximize their overall revenue by considering the following subproblem shown in Eq. (16) in each time slot $t$.

$$\max R_i^{t'} = \frac{p_i^{t'}}{r_i^{t'}} \quad (16)$$

subject to $x_i^{t'} \in D^r$

The above problem can be solved by considering the game played by NSP $S_1$ and $S_2$. The Nash Equilibrium point is the solution of the problems. Now we consider that two NSPs play a Bertrand competition (or price competition) game in each time slot $t$. The Bertrand competition game $\Gamma(\text{Player, Strategy, Payoff})$, is described as follows:

- **Player:** The NSP $S_1$ and $S_2$ are the two players in the game.
- **Strategy**: The strategy is the price set by NSP \(S_i\) for \(i = 1, 2\).
- **Payoff**: The payoff is the revenue get by NSP \(S_i\) for \(i = 1, 2\).

In this game, NSP \(S_1\) and \(S_2\) set their price \(p_1^*\) and \(p_2^*\) respectively, to get market share to maximize their revenue, which is the multiplication of price and the market share (or the number of users). The number of users in each NSP’s network can be derived by Proposition 1. In order to derive the number of users in each time slot, both NSPs should know the QoS function of each other. The QoS function is predetermined by the technology and capacity investment. For example, the QoS functions of 3G network and 4G network are different. The QoS function of each NSP \((h_i(\cdot))\) is the same for all the time slot. We assumed that each NSP observes the QoS function of the other NSP by investigating the technology and capacity investment before the game of the first time slot. Before playing the game in each time slot, we use the number of users in the last time slot \((x_t^i)\) to estimate that in the current slot \((x_t^i)\).

If the QoS function of one NSP can not be derived by the other NSP, namely, incomplete information, a Bayesian game [10] can be employed to modeling the interaction between the two NSPs. And Bayesian Nash equilibrium need to establish. This will be investigated in the next Section.

**Lemma 2**: The condition for existence of the Nash Equilibrium of the game \(\Gamma(\text{Player, Strategy, Payoff})\) is

\[
\varphi' > \frac{p_1^*}{h_1(x_t^i)} \geq \frac{p_2^*}{h_2(x_t^i)} > 0
\]  
(17)

**Proof.** As illustrated in Proposition 1, the price per QoS determines the market share of a NSP. If \(\frac{p_1^*}{h_1(x_t^i)} < \frac{p_2^*}{h_2(x_t^i)}\), by Proposition 1, the number of users in NSP \(S_2\)’s network is 0, thus the payoff of NSP \(S_2\) is 0, while the payoff the NSP \(S_1\) is positive. Therefore, the price strategy \(p_2^*\) in this case is a dominated strategy for NSP \(S_2\). NSP \(S_1\) would not play the dominated strategy. In order to get a positive market share, NSP \(S_2\) would decrease its price per QoS, which lead to the condition \(\frac{p_1^*}{h_1(x_t^i)} \geq \frac{p_2^*}{h_2(x_t^i)}\). By Proposition 1, both NSP \(S_1\) and \(S_2\) have positive number of users in their network. Both NSP \(S_1\) and \(S_2\) can get positive payoff.

Q.E.D.

**Proposition 2**: If the following conditions are satisfied,

1. Users’ QoS valuation is distributed uniformly.
2. NE exists, which means that the condition in Lemma 2 is satisfied.
3. The QoS function has the following property, shown in inequality (18),

\[
h_2(x_t^i) < 4
\]

(18)

then, the Nash Equilibrium of the game \(\Gamma(\text{Player, Strategy, Payoff})\) is unique, and the NE point \((p_1^{\ast}, p_2^{\ast})\) can be expressed as Eq. (19) and Eq. (20).

\[
p_1^{\ast} = \frac{2\varphi'(h_1(x_t^i) - h_2(x_t^i))h_1(x_t^i)}{4h_1(x_t^i) - h_2(x_t^i)}
\]
(19)

\[
p_2^{\ast} = \frac{\varphi'(h_1(x_t^i) - h_2(x_t^i))h_2(x_t^i)}{4h_1(x_t^i) - h_2(x_t^i)}
\]
(20)

**Proof.** Please refer to Appendix C for the details of proof.

**Proposition 3**: If the conditions in Proposition 2 are satisfied, the upper-bounds of the prices of NSP \(S_1\) and \(S_2\) at NE are \(2\varphi' h_1(x_t^i)\) and \(\varphi' h_2(x_t^i)\), respectively.

**Proof.** Since the price of NSP \(S_1\) at NE can be expressed as Eq. (19), we have

\[
p_1^{\ast} = \frac{2\varphi'(h_1(x_t^i) - h_2(x_t^i))h_1(x_t^i)}{4h_1(x_t^i) - h_2(x_t^i)}
\]

\[
< \frac{2\varphi'(h_1(x_t^i) - h_2(x_t^i))h_1(x_t^i)}{h_1(x_t^i) - h_2(x_t^i)}
\]

\[
= 2\varphi' h_1(x_t^i)
\]

(21)

Therefore, the upper-bound of the price of NSP \(S_1\) at NE is \(2\varphi' h_1(x_t^i)\).

Similarly, we can derive the upper-bound of the price of NSP \(S_2\) at NE by Eq. (20) as the follows:

\[
p_2^{\ast} = \frac{\varphi'(h_1(x_t^i) - h_2(x_t^i))h_2(x_t^i)}{4h_1(x_t^i) - h_2(x_t^i)}
\]

\[
< \frac{\varphi'(h_1(x_t^i) - h_2(x_t^i))h_2(x_t^i)}{h_1(x_t^i) - h_2(x_t^i)}
\]

\[
= \varphi' h_2(x_t^i)
\]

(22)

Q.E.D.

According to the Proposition 3, we can see that the upper-bounds of the prices of NSP \(S_1\) and NSP \(S_2\) at NE are functions of \(h_1(x_t^i)\) and \(h_2(x_t^i)\), respectively. Therefore, we can analyze the impact of QoS function on the prices at NE. We can conclude that the larger the value of QoS is, the larger the price at NE is. Additionally, the upper-bounds are also functions of \(\varphi'\). This indicates that the larger the upper-bound of users’ valuation on QoS is, the larger the price at NE is.

**Proposition 4**: If the conditions in Proposition 2 are satisfied, the ratio of revenue of NSP \(S_1\) to that of NSP \(S_2\) at NE can be expressed as Eq. (23)

\[
\frac{R_1^c}{R_2^c} = \frac{2h_1(x_t^i)}{h_2(x_t^i)}
\]

(23)

**Proof.** Please refer to Appendix D for the details of proof.

According to Proposition 4, we can analyze the impact of QoS function on the revenue of NSP. Equation (23) shows that ratio of revenue of NSP \(S_1\) to that of NSP \(S_2\) is proportional to the ratio of QoS level of NSP \(S_1\) to that of NSP
S2. The relative level of NSP’s QoS determines its revenue share. However, whether the NSP provides the better QoS or not is determined by the profit (Please note that in economics: profit=revenue-cost.) that an NSP can get. In other words, the investment cost should also be considered. It is left to the future work to discuss the NSP’s choice of QoS function (or network upgrade) by jointly considering the revenue and investment cost.

5. Simulation

This section makes simulation analysis to validate our analytical results. The simulation analysis includes the following aspects:

- Compare the revenue from TDP scheme with the revenue from Time-Independent Pricing(TIP) scheme for the duopoly case.
- Compare the number of users for TDP scheme with the number of users for TIP scheme for the duopoly case.
- Compare the total revenue gets by NSP S1 and S2 with that from monopoly NSP.
- Evaluate revenue of NSPs for various QoS functions.

Please refer to Appendix E for the analysis in the case of monopoly NSP.

The parameters for simulations are summarized in Table 2. We assume that the distribution of the users’ valuation $\theta_t^k$ of QoS follows an uniform distribution $[0, \varphi^k]$. When $t \in [1, 8]$ or $t \in [17, 24]$, $\varphi^k = 2$, and when $t \in [9, 16]$, $\varphi^k = 4$. Therefore, the users averagely have much higher valuation of QoS during time slots [9, 16] than that during other time slots. It can be expected that the peak traffic will occur during time slots [9, 16]. Three groups of QoS functions $(H(a), H(b), H(c))$ are used in the simulation. The QoS function is defined as simple affine function satisfying Assumption 1 aforementioned. This kind of affine QoS function has been also adopted in [17], and also satisfies the conditions in Proposition 2. The two NSPs set the prices in each time slot according to the NE established in Proposition 2.

**Observation 1:** For each NSP, the revenue is more stable in the TDP scheme than that in TIP scheme.

We can see from Fig. 2(a) and Fig. 2(b) that, the revenue of NSP is oscillated in the TIP scheme. In the TIP scheme, the price is fixed initially, then the number of users in a network (for example, network of $S_1$) with smaller price per QoS keeps increasing, which makes the network of $S_1$ congested. The users who get negative payoff in network of $S_1$ due to congestion will switch to network of $S_2$, which makes the number of users in network of $S_1$ decreases. The above process happens iteratively, which makes the revenue oscillated in TIP scheme. However, in the TDP scheme, NSPs can use price as a congestion management tool to control the number of users choosing their network at each time slot.

**Observation 2:** The revenue from TDP scheme is higher than that from the TIP scheme in the duopoly competition environment.

In TDP scheme, NSPs can set new prices for each time slot, which is the Nash Equilibrium of the game $\Gamma$, thus the revenue of NSPs get maximized at each time slot. However, in TIP scheme, the price can only be set initially without considering the competition from the rival NSP and the congestions.
Fig. 3 Comparison of the proportion of users in network of NSP $S_i$ under the TIP and TDP schemes with QoS function group H(b).

Fig. 4 The price of NSP $S_1$ and NSP $S_2$ under TDP with QoS function group H(b).

Fig. 5 Comparison of total revenue from monopoly NSP and duopoly NSPs under the TDP scheme with QoS function group H(b).

**Observation 3:** TDP scheme has congestion control effect in the duopoly case.

We can see from Fig. 3, in the NSP $S_1$’s network, the number of users in “peak hours” under TIP scheme is much more than that of number of users in “peak hours” under TDP Scheme. The reason is that when the price is time-dependent, the NSPs can increase its price to make less users to use its network. It is interesting to see that, the number of users in “peak hours” under TIP scheme is much less than the number of users in “peak hours” under TDP scheme for NSP $S_2$’s network. The reason is that the QoS provided by network of NSP $S_2$ is lower than that of NSP $S_1$ (see Assumption 1). Under TIP scheme, high QoS valuation users tend to choose network of NSP $S_1$. However, under TDP scheme, the competition effect push the users from NSP $S_1$’s network to NSP $S_2$’s network.

Under TIP scheme, another interesting point is that the number of users in both two NSPs’ networks heavily changes from $t=0$ to $t=8$ and from $t=17$ to $t=24$, while the number of users in both two NSPs’ networks do not changes obviously from $t=9$ to $t=16$. From $t=0$ to $t=8$ and from $t=17$ to $t=24$, when the NSP $S_1$ and $S_2$ fixed their price in all time slot, the NSP with lower price per QoS (for example NSP $S_1$) attracts users in time slot $t$. However, the increased number of users will deteriorate the QoS of the NSP $S_1$’s network, which lead to a much higher price per QoS of NSP $S_1$ in time slot $t+1$. Then users will switch to the network of NSP $S_2$ in time slot $t+1$. Similarly, the number of users in NSP $S_2$’s network will increase in time slot $t+1$, which will deteriorate the QoS of the NSP $S_2$’s network. This makes price per QoS of network $S_2$ much higher. Users will switch to NSP $S_1$’s network in time slot $t+2$. Recursively, a kind of oscillation phenomena appears. However, the reason why the oscillation phenomena is not obvious from $t=9$ to $t=16$ is that the upper bound of users’ QoS valuation (i.e., $\varphi'=4$) is much higher than that of other time slots (i.e., $\varphi'=2$). Even if the number of users of one NSP’s network increases and the QoS deteriorates, users may not choose to switch to another network due to users’ high QoS valuation.

**Observation 4:** The revenue from TDP scheme in the duopoly NSP case is smaller than that in a monopoly NSP case.

In the monopoly case, the NSP has market power, all the surplus can be extracted by the NSP. However, due to the competition effect, all the surplus cannot be extracted by the duopoly NSPs, a part of the surplus goes to users. Thus, The revenue from TDP scheme in the duopoly NSP case is smaller than that in a monopoly NSP case. Please refer to Fig. 5.

**Observation 5:** The relative level of NSP’s QoS determines its revenue share.

In order to evaluate the impact of QoS function on the NSP revenue, we have done simulation with three groups of QoS functions (Please see Table 2). We denote the revenue...
of NSP $S_1$ with QoS function group H(k) (where k=a, b, c) as $R_{k}^{H(k)}$. Figure 6 shows that $R_{1}^{H(c)} > R_{1}^{H(b)} > R_{1}^{H(a)}$, and $R_{2}^{H(c)} < R_{2}^{H(b)} < R_{2}^{H(a)}$. We can also find from Table 2 that the ratio of $h_1$ to $h_2$ of QoS function group H(a) is less than that of QoS function group H(b), and the ratio of $h_1$ to $h_2$ of QoS function group H(c) is less than that of QoS function group H(b). The relative level of QoS of NSP $S_1$ with QoS function group H(c) is the largest, therefore, the revenue share of $S_1$ with QoS function group H(c) is the largest. The relative level of QoS of NSP $S_2$ with QoS function group H(a) is the smallest, therefore, the revenue share of NSP $S_2$ with QoS function group H(a) is the smallest. These results are consistent with the Proposition 4.

Another interesting alternative is the asymmetrical case that one NSP uses TDP scheme, and the other NSP uses TIP scheme. The Bertrand game is no longer suitable for modeling this asymmetrical case. However, some hints can also get from the Bertrand game. For example, the NSP with lower price per QoS has competitive advantage over the NSP with relative higher price per QoS. Therefore, the NSP with TDP scheme can change his price strategy over the time slots to keep competitive advantage, while the NSP with TIP cannot. The NSP with TDP scheme is still expected to get much higher revenue than that from TIP scheme.

For rigorous analysis of the asymmetrical case, a Stackelberg game can be used to model the asymmetrical case. The leader of Stackelberg game is the NSP with TIP scheme, while the NSP with TDP scheme is the follower. In the first stage of the Stackelberg game, the NSP with TIP scheme set a fixed price, then in the second stage of the Stackelberg game, the NSP with TDP scheme can observe the leader’s price strategy and set time-dependent price to maximize his own revenue. Backward induction can be used to solve the NE of the Stackelberg game in which the second stage problem of the game is solved firstly, given the fixed price of leader. Then the first stage problem of the Stackelberg game can be solved. The specific analysis of the Stackelberg game model is left to the future work.

6. Conclusion

This paper analyzes the time-dependent pricing scheme in a duopoly competition environment. We model the NSP duopoly competition as a Bertrand competition (price competition) game, in which each NSP sets time-dependent price to compete for market share (number of users) to maximize its revenue. The sufficient condition for existence of the Nash equilibrium of the Bertrand is established. Unique Nash equilibrium is also established under the assumption that the users’ valuation of QoS is uniformly distributed. The simulation results reflect that the revenue from a time-dependent pricing scheme is higher that from the time-independent pricing scheme in the duopoly case. However, due to competition effect, the NSPs could not extract all the surplus from users. In this sense, we can conclude that the time-dependent pricing scheme in a competitive environment can also benefit NSPs, but the revenue improvement is limited due to competition effect.

This work assumes that users’ switching cost is zero, but it can be relaxed by considering the switching cost as a part of the price set by the NSPs, and then the analytical results from this paper are still valid. However, the non-zero switching cost can cause a “lock-in” effect for users. It would be interesting to relax this assumption to see the impact of the “lock-in” effect on the NSPs’ revenue in the future work. Future work should also include an examination of the effect of oligopoly competition between NSPs. For example, if this model is extend to three NSPs (tripoly) case, it is considered that the key ideas are also valid. One is that the price per QoS determine market share of NSP. If a NSP can provide lower price per QoS, its market share is expected to increase. However, it is very difficult to get the explicit analytical result for this case since the marginal points of users’ QoS valuation that determine the users’ choice of network will increase dramatically.

References

According to Eq. (9) and Eq. (10), we have

\[ r_{S_2}^0 - r_{S_1}^0 = \frac{p_{S_2}^0}{h_2(x_{S_2}^{t-1})} - \frac{p_{S_1}^0}{h_1(x_{S_1}^{t-1})} = \frac{p_{S_2}^0 h_1(x_{S_1}^{t-1}) - p_{S_1}^0 h_2(x_{S_2}^{t-1})}{h_1(x_{S_1}^{t-1}) h_2(x_{S_2}^{t-1})} \quad (A-1) \]

According to Eq. (9), Eq. (11) and Eq. (A-1), we have

\[ r_{k}^1 - r_{S_2}^0 = \frac{p_{S_2}^1}{h_2(x_{S_2}^{t-1})} - \frac{p_{S_1}^1}{h_1(x_{S_1}^{t-1})} = \frac{p_{S_2}^1 h_1(x_{S_1}^{t-1}) - p_{S_1}^1 h_2(x_{S_2}^{t-1})}{h_1(x_{S_1}^{t-1}) h_2(x_{S_2}^{t-1})} \quad (A-2) \]

Similarly, according to Eq. (10), Eq. (11) and Eq. (A-1), we have

\[ r_{S_2}^1 - r_{S_1}^0 = \frac{-h_1(x_{S_1}^{t-1})}{h_1(x_{S_1}^{t-1}) - h_2(x_{S_2}^{t-1})} (r_{S_2}^0 - r_{S_1}^0) \quad (A-3) \]

Due to Assumption 1, we have \( h_2(x_{S_2}^{t-1}) > 0 \) and \( h_1(x_{S_1}^{t-1}) > h_2(x_{S_2}^{t-1}) \), then

\[ \frac{-h_1(x_{S_1}^{t-1})}{h_1(x_{S_1}^{t-1}) - h_2(x_{S_2}^{t-1})} < 0 \quad (A-4) \]

If \( \frac{p_{S_2}^0}{h_2(x_{S_2}^{t-1})} < \frac{p_{S_1}^0}{h_1(x_{S_1}^{t-1})} \), or \( r_{S_2}^0 > r_{S_1}^0 \), by Eq. (A-2) and Eq. (A-4), we have the following result:

\[ r_{k}^1 - r_{S_2}^0 < 0 \quad (A-5) \]

Therefore, we have \( r_{k}^1 < r_{S_2}^0 < r_{S_1}^0 \).

Similarly, if \( \frac{p_{S_2}^0}{h_2(x_{S_2}^{t-1})} \geq \frac{p_{S_1}^0}{h_1(x_{S_1}^{t-1})} \), by Eq. (A-3) and Eq. (A-4), we have the following result:

\[ r_{k}^1 - r_{S_1}^0 \geq 0 \quad (A-6) \]

Therefore, \( r_{k}^1 \geq r_{S_2}^0 \geq r_{S_1}^0 \).

**Appendix B: Proof of Proposition 1**

1. If \( \frac{p_{S_2}^0}{h_2(x_{S_2}^{t-1})} < \frac{p_{S_1}^0}{h_1(x_{S_1}^{t-1})} \), when \( \theta_k^0 > r_{S_1}^0 \), by Lemma 1, we have \( \theta_k^0 > r_{S_1}^0 > r_{S_2}^0 \). According to Eq. (7) and Eq. (8), we can get \( u_{k,1}^0 > u_{k,2}^0 \), this means that the user with \( \theta_k^0 > r_{S_1}^0 \) will choose NSP \( S_1 \). By Assumption 2, we can calculate the number of user who choose NSP \( S_1 \) by

\[ \Omega_1(x^{t-1}) = P(\theta_k^0 > r_{S_1}^0) = 1 - P(\theta_k \leq r_{S_1}^0) = 1 - F(x^{t-1}) \quad (A-7) \]

However, when \( \theta_k^0 < r_{S_1}^0 \), by Lemma 1, we have \( \theta_k^0 < r_{S_1}^0 < r_{S_2}^0 \), therefore, user \( k \) get negative payo from choosing either NSP \( S_1 \) or NSP \( S_2 \).

We illustrate the users’ choice in this case in Fig. A.1.

2. If \( \frac{p_{S_2}^0}{h_2(x_{S_2}^{t-1})} \geq \frac{p_{S_1}^0}{h_1(x_{S_1}^{t-1})} \), when \( \theta_k^0 > r_{S_1}^0 \), by Lemma 1, we have \( \theta_k^0 > r_{S_1}^0 \geq r_{S_2}^0 \). According to Eq. (7) and Eq. (8), we can get \( u_{k,1}^0 > u_{k,2}^0 \), this means that the user with \( \theta_k^0 > r_{S_1}^0 \) will choose NSP \( S_1 \). By Assumption 2, we can calculate the number of user who choose NSP \( S_1 \) by

\[ \Omega_1(x^{t-1}) = P(\theta_k^0 > r_{S_1}^0) = 1 - P(\theta_k \leq r_{S_1}^0) = 1 - F(x^{t-1}) \quad (A-8) \]
When \( \tau_k^0 \geq \theta_k^0 \geq \tau_{S_2}^0 \), according to Eq. (7) and Eq. (8), we can get \( u_{k,1}^* < u_{k,2}^* \); this means that the user will choose NSP \( S_2 \). We can calculate the number of user who choose NSP \( S_2 \) by

\[
\Omega_2(x^*) = P(\tau_k^1 \geq \theta_k^0 \geq \tau_{S_2}^0) = \int_{\tau_{S_2}^0}^{\tau_k^1} f(y)dy = F(\tau_k^1) - F(\tau_{S_2}^0) \quad (A\cdot9)
\]

We illustrate the users’ choice in this case in Fig. A-2.

**Appendix C: Proof of Proposition 2**

By condition (1), we have \( F'(\theta_k^0) = \frac{\theta_k^0}{2} \). Since the NE is exist, we can express the revenue of NSP \( S_1 \) as the following (by Proposition 1),

\[
R_1^* = \Omega_1(x^*) \cdot p_1^* = \left[ 1 - F'(\tau_k^1) \right] \cdot p_1^* = \left[ 1 - \frac{p_1^* - p_2^*}{\varphi(h_1(x_1^1) - h_2(x_2^1))} \right] p_1^* \quad (A\cdot10)
\]

To maximize \( R_1^* \), we have the following optimal condition,

\[
\frac{dR_1^*}{dp_1^*} = 0 \quad (A\cdot11)
\]

Therefore, we can get the optimal price by solving the Eq. (A-11),

\[
p_1^* = BR_1(p_2^*) = \frac{1}{2} \left[ \varphi(h_1(x_1^1) - h_2(x_2^1)) + p_2^* \right] \quad (A\cdot12)
\]

The optimal price of NSP \( S_1 \) is a function of \( p_2^* \), which is defined as function \( BR_1(p_2^*) \). In game theory [10], we call the function \( BR_1(p_2^*) \) as best response function of player NSP \( S_1 \).

Similarly, the revenue of NSP \( S_2 \) can be expressed as

\[
R_2^* = \Omega_2(x^*) \cdot p_2^* = \left[ F'(\tau_k^1) - F'(\tau_{S_2}^0) \right] \cdot p_2^* = \left[ \frac{p_1^* - p_2^*}{\varphi(h_1(x_1^1) - h_2(x_2^1))} - \frac{p_2^*}{\varphi_2(x_2^1) - h_2(x_2^1)} \right] p_2^* \quad (A\cdot13)
\]

To maximize \( R_2^* \) by letting \( \frac{dR_2^*}{dp_2^*} = 0 \), we can get the best response function of player NSP \( S_2 \).

\[
p_2^* = BR_2(p_1^*) = \frac{h_2(x_2^1)}{2h_1(x_1^1)} \cdot p_1^* \quad (A\cdot14)
\]

We have drawn the best response function in Fig. A-3. The condition (3) in this proposition means that the slopes of \( BR_1(p_2^*) \) and \( BR_2(p_1^*) \) satisfy \( \frac{h_1(x_1^1)}{h_2(x_2^1)} < 1 \) which ensure that the \( BR_1(p_2^*) \) and \( BR_2(p_1^*) \) intersect at a point \( B \), which is the unique NE of the game (Player, Strategy, Payoff).

By combining Eq. (A-12) and Eq. (A-14), the NE point \( (p_1^*, p_2^*) \) can be solved and expressed as follows:

\[
p_1^* = \frac{2\varphi(h_1(x_1^1) - h_2(x_2^1))h_1(x_1^1)}{4h_1(x_1^1) - h_2(x_2^1)} \quad (A\cdot15)
\]

\[
p_2^* = \frac{\varphi(h_1(x_1^1) - h_2(x_2^1))h_2(x_2^1)}{4h_1(x_1^1) - h_2(x_2^1)} \quad (A\cdot16)
\]

**Appendix D: Proof of Proposition 4**

By substituting the NE prices Eq. (A-15) and Eq. (A-16) into Eq. (A-10), we can derive the revenue of NSP \( S_1 \) at NE point as the follows:

\[
R_1^* = \left[ 1 - \frac{p_1^* - p_2^*}{\varphi(h_1(x_1^1) - h_2(x_2^1))} \right] p_1^* = \left[ 1 - \frac{2\varphi(h_1(x_1^1) - h_2(x_2^1))h_1(x_1^1)}{4h_1(x_1^1) - h_2(x_2^1)} \right] p_1^* = \left[ 1 - \frac{\varphi(h_1(x_1^1) - h_2(x_2^1))h_1(x_1^1)}{4h_1(x_1^1) - h_2(x_2^1)} \right] p_1^* \quad (A\cdot17)
\]

Similarly, by substituting the NE prices Eq. (A-15) and
Eq. (A-16) into Eq. (A-13), we can derive the revenue of NSP $S_2$ at NE point as the follows:

$$R^*_2 = \left[ p_2^s - p_2^* \right] \left[ \frac{\varphi'(h_1(x_1^s) - h_2(x_2^s))}{\varphi'(h_2(x_2^s))} \right] - p_2^* \left[ \frac{\varphi'(h_2(x_2^s))}{\varphi'(h_2(x_2^s))} \right] \left[ \frac{2\varphi'(h_1(x_1^s) - h_2(x_2^s))]}{[4h_1(x_1^s) - h_2(x_2^s)]^2} \right] \left[ \frac{2\varphi'(h_1(x_1^s) - h_2(x_2^s))]}{[4h_1(x_1^s) - h_2(x_2^s)]^2} \right]$$

(A-18)

By combining Eq. (A-17) and Eq. (A-18), we can get the ratio of revenue of NSP $S_1$ to that of NSP $S_2$ as in Eq. (A-19)

$$\frac{R^*_1}{R^*_2} = \frac{4\varphi'[h_1(x_1^s)](h_1(x_1^s) - h_2(x_2^s))}{4h_1(x_1^s) - h_2(x_2^s)]^2}$$

(A-19)

Appendix E: Analysis for the case of monopoly NSP

We analyze the monopoly NSP case for comparison. Suppose that there is only a monopoly NSP $S_1$ in the market, other assumptions are the same as the duopoly case. Different from the duopoly case, $\tau^0_{S_1}$ denote the only marginal point where users switch from getting negative payoff to deriving positive payoff from choosing NSP $S_1$, i.e., for a give $x'$, $\tau^0_{S_1}$ is a point such that $u'_{k,1} = 0$. We have

$$u'_{k,1} = \theta h_1(x') - p'_1 > 0 \text{ if } \theta > \tau^0_{S_1} \quad (A-20)$$

Eq. (A-20) means that user with valuation of QoS greater than $\tau^0_{S_1}$ will use network of NSP $S_1$. The number of user who use network of NSP $S_1$ is

$$x'_1 = P(\theta > \tau^0_{S_1}) = 1 - P(\theta \leq \tau^0_{S_1}) = 1 - F(\tau^0_{S_1}) \quad (A-21)$$

By setting $u'_{k,1} = 0$, we get,

$$p'_1 = \tau^0_{S_1} h_1(x'_1) = \tau^0_{S_1} h_1(1 - F(\tau^0_{S_1})) \quad (A-22)$$

Therefore, the revenue maximization problem of the monopoly NSP $S_1$ is

$$\max_{p'_1} R^*_1(p'_1) = \max_{p'_1} p'_1 x'_1 \quad \max_{\tau^0_{S_1}} \tau^0_{S_1} h_1(1 - F(\tau^0_{S_1}))(1 - F(\tau^0_{S_1})) \quad (A-23)$$

Cheng Zhang received his B.S. degree in Automation from Wuhan University of Science and Technology, Wuhan, China, in 2005 and M.S. degree in Control Theory and Control Engineering from Shanghai Jiao Tong University, Shanghai, China, in 2008. He is currently an invited researcher at Global Information and Telecommunication Institute, Waseda University. His research interests include congestion-based pricing and game theory application in telecommunication network. He received the IEICE Young Researcher’s Award in 2013.

Bo Gu received the B.E. degree from Tianjin University, Tianjin, China, in 2004, M.E. degree from Peking University, Beijing, China, in 2007, and Ph.D. degree from Waseda University, Tokyo, Japan, in 2013, respectively. From 2007 to 2011, he was a research engineer at Sony Digital Network Applications, Japan. He is currently an assistant professor at School of Fundamental Science and Engineering, Waseda University. His research interests include resource and mobility management of wireless networks, QoS provisioning, and next generation wireless networks. He received the IEICE Young Researcher’s Award in 2011, and the IEICE Communication Quality Conference Premium Award in 2012.

Kyoko Yamori received her B.A. degree in business administration and M.A. and Ph.D. degrees in information management science from Asahi university in 1995, 1997, 2000. She is presently an associate professor at Department of Management Information, Asahi University, and a visiting associate professor at Global Information and Telecommunication Institute, Waseda University. She received the IEICE Switching System Research Award in 2001, the IEICE Young Researcher’s Award in 2005, and the IEICE Best Paper Award in 2005.

Sugang Xu received his B.E. and M.E. degrees in computer engineering from Beijing Polytechnic University, Beijing, China, in 1994 and 1997, respectively, and Ph.D. degree in information and communication engineering at the University of Tokyo, Tokyo, Japan, in 2002. He joined Global Information and Telecommunication Institute, Waseda University in 2002, as a research associate there. Since 2005, he joined National Institute of Information and Communications Technology (NICT), Tokyo, Japan, as an expert researcher. He is also an invited researcher at Waseda University. His research interests include algorithms, network architectures, photonic network control, optical grid network systems, parallel and distributed processing. He is a member of IEEE.
Yoshiaki Tanaka received the B.E., M.E., and D.E. degrees in electrical engineering from the University of Tokyo, Tokyo, Japan, in 1974, 1976, and 1979, respectively. He became a staff at Department of Electrical Engineering, the University of Tokyo, in 1979, and has been engaged in teaching and researching in the fields of telecommunication networks, switching systems, and network security. He was a guest professor at Department of Communication Systems, Lund Institute of Technology, Sweden, from 1986 to 1987. He was also a visiting researcher at Institute for Posts and Telecommunications Policy, from 1988 to 1991, and at Institute for Monetary and Economic Studies, Bank of Japan, from 1994 to 1996. He is presently a professor at Global Information and Telecommunication Institute, Waseda University, and a visiting professor at National Institute of Informatics. He received the IEEE Outstanding Student Award in 1977, the Niwa Memorial Prize in 1980, the IEICE Achievement Award in 1980, the Okawa Publication Prize in 1994, the TAF Telecom System Technology Award in 1995 and in 2006, the IEICE Information Network Research Award in 1996, in 2001, in 2004, and in 2006, the IEICE Communications Society Activity Testimonial in 1997, in 1998, the IEICE Switching System Research Award in 2001, the IEICE Best Paper Award in 2005, the IEICE Network System Research Award in 2006, in 2008, and in 2011, the IEICE Communications Society Activity Award in 2008, the Commendation by Minister for Internal Affairs and Communications in 2009, the APNOMS Best Paper Award in 2009 and in 2012, and the IEICE Distinguished Achievement and Contributions Award in 2013.